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# Predictions for Lepton Flavour Violation in $Z$ decays<sup>§</sup>

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Recent experimental results suggest that the neutrinos of the Standard Model are massive, though light. Therefore they may mix with each other giving rise to lepton flavour or even lepton number violating processes, depending on whether they are Dirac or Majorana particles. Furthermore, the lightness of the observed neutrinos may be explained by the existence of heavy ones, whose effects on LFV would be very sizeable. We present an analysis of the effect of massive neutrinos on flavour-changing decays of the  $Z$  boson into leptons, at the one-loop level, independent of neutrino mass models. Constraints from present experiments are taken into account.

## 1. Introduction and motivation

With the Giga- $Z$  option of the Tesla linear collider project one may expect the production of about  $10^9$   $Z$  bosons at resonance [1]. This huge rate, about a factor 100 higher than at LEP 1, allows one to study a number of problems with unprecedented precision. Among them is the search for lepton-flavour changes in  $Z$  decays:  $Z \rightarrow e\mu, \mu\tau, e\tau$ . The best *direct* limits are obtained by searches at LEP 1 (95% c.l.) [2]:

$$\text{BR}(Z \rightarrow e^\mp \mu^\pm) < 1.7 \times 10^{-6}, \quad (1)$$

$$\text{BR}(Z \rightarrow e^\mp \tau^\pm) < 9.8 \times 10^{-6}, \quad (2)$$

$$\text{BR}(Z \rightarrow \mu^\mp \tau^\pm) < 1.2 \times 10^{-5}. \quad (3)$$

A careful analysis shows that the sensitivities could be largely improved at the Giga- $Z$  [3],

$$\text{BR}(Z \rightarrow e^\mp \mu^\pm) < 2 \times 10^{-9}, \quad (4)$$

$$\text{BR}(Z \rightarrow e^\mp \tau^\pm) < f \times 6.5 \times 10^{-8}, \quad (5)$$

$$\text{BR}(Z \rightarrow \mu^\mp \tau^\pm) < f \times 2.2 \times 10^{-8}, \quad (6)$$

with  $f = 0.2 \div 1.0$ .

Non-zero rates are expected if neutrinos are massive and mix [4]. From experiments we have evidence of tiny neutrino masses and substantial mixings. Unnaturally small mass scales may be indicative for a mechanism which produces at the

same time very large masses. Heavy neutrinos are introduced in most GUTs [5] and string-inspired models [6], and are suggested by the seesaw mechanism [7].

The above observations motivate us to have a closer look at the prospects of observing LFV. We will explore here the following scenarios:

(i) The  $\nu\text{SM}$ . We treat the known  $n_G = 3$  generations of *light neutrinos* ( $\nu_e, \nu_\mu, \nu_\tau$ ) as *massive Dirac* particles. Individual lepton numbers  $L_e, L_\mu, L_\tau$  are not conserved (in analogy to the quark sector). As a by-product, the  $Z$  decay amplitude into two quarks of different flavours can be read off from our general expressions.

(ii) The  $\nu\text{SM}$  *extended* with *one heavy ordinary Dirac* neutrino (usual  $\text{SU}(2)_L \otimes \text{U}(1)_Y$  quantum numbers). This case implies the existence of a heavy charged lepton as well. It is a simple application of case (i) for heavier neutrinos. Again, total lepton number  $L$  is conserved.

(iii) The  $\nu\text{SM}$  *extended* with  $n_R = 2$  *heavy right-handed singlet Majorana* neutrinos. Not only individual, but also total  $L$  is, in general, not conserved since the presence of Majorana mass terms involves mixing of neutrinos and antineutrinos, with opposite lepton number. For two equal and heavy masses this case reduces to the addition of *one heavy singlet Dirac* neutrino. In this latter case  $L$  is recovered [8].

Our results are independent of neutrino mass models. We take into account constraints on neutrino masses and mixings given by oscillation ex-

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periments (light sector) or imposed by unitarity and precision tests of flavour diagonal and nondiagonal processes (heavy sector).

## 2. $Z \rightarrow \ell_1 \ell_2$ with massive neutrinos

The amplitude for the decay of a  $Z$  boson into two charged leptons with different flavour ( $\ell_1$  and  $\ell_2$ ) vanishes in Born approximation but receives quantum corrections. It can be written for massless external fermions as:

$$\mathcal{M} = -\frac{ig\alpha_W}{16\pi c_W} \mathcal{V} \varepsilon_Z^\mu \bar{u}_{\ell_2}(p_2) \gamma_\mu (1 - \gamma_5) u_{\ell_1}(-p_1) \quad (7)$$

with  $\alpha_W = \alpha/s_W^2$ . The dimensionless form factor  $\mathcal{V}$  is a function of  $\lambda_Q \equiv Q^2/M_W^2$ , with  $Q^2 = (p_2 - p_1)^2$  (to be fixed at the  $Z$  peak), and the masses  $\lambda_i \equiv m_i^2/M_W^2$  and mixings of the massive, virtual neutrinos  $i$  [9,10].  $\mathcal{V}$  gets contributions from one-loop vertex- and self-energy-graphs, that are calculated here in the 't Hooft-Feynman gauge. The former consist of triangle graphs: two virtual neutrinos coupled to the external  $Z$  boson with a  $W$  or a Goldstone boson  $\phi$  being exchanged ( $W, \phi$ ); and one virtual neutrino exchanged with  $W$  or  $\phi$  coupled to the  $Z$  ( $WW, \phi\phi$  and  $W\phi$ ). The self-energy graphs are corrections to the external fermion legs ( $\Sigma$ ).

One must discuss separately the cases of *ordinary* Dirac and *general* Majorana neutrinos [11]:

Dirac:

$$\mathcal{V}_D = \sum_{i=1}^{n_G} \mathbf{V}_{\ell_1 i} \mathbf{V}_{\ell_2 i}^* V(i), \quad (8)$$

$$V(i) = v_W(i) + v_\phi(i) + v_{WW}(i) + v_{\phi\phi}(i) + v_{W\phi}(i) + v_\Sigma(i), \quad (9)$$

Majorana:

$$\mathcal{V}_M = \sum_{i,j=1}^{n_G+n_R} \mathbf{B}_{\ell_1 i} \mathbf{B}_{\ell_2 j}^* V_M(i, j), \quad (10)$$

$$V_M(i, j) = v_W(i, j) + v_\phi(i, j) + v_{WW}(i) + v_{\phi\phi}(i) + v_{W\phi}(i) + v_\Sigma(i), \quad (11)$$

where only global mixing factors are extracted: the *leptonic CKM* mixing matrix  $\mathbf{V}$  for ordinary Dirac neutrinos, and its *generalized* version,  $\mathbf{B}$ ,

for Majorana neutrinos. The latter mixings appear in charged-current interactions of the left-handed (LH) components of  $n_G$  generations of ordinary charged leptons ( $\ell_L^0 = e_L, \mu_L, \tau_L, \dots$ ) with  $n_G$  LH isodoublet neutrinos ( $\nu_L^0 = \nu_e, \nu_\mu, \nu_\tau, \dots$ ). The interaction eigenstates are in general not the same as the physical mass eigenstates but a mixture [12–14],

$$\ell_{Li}^0 = \sum_{j=1}^{n_G} \mathbf{U}_{ij}^{\ell_L} \ell_{Lj}, \quad \nu_{Li}^0 = \sum_{j=1}^{n_G+n_R} \mathbf{U}_{ij} \nu_{Lj}, \quad (12)$$

where  $\nu = \eta \nu^c$  are  $n_G + n_R$  Majorana fields (self-conjugate up to a phase  $\eta$ ). In the physical basis,

$$\begin{aligned} -\mathcal{L}_{CC} &= \frac{g}{\sqrt{2}} W_\mu \bar{\ell}_{Li}^0 \gamma^\mu P_L \nu_{Li}^0 + h.c. \\ &= \frac{g}{\sqrt{2}} W_\mu \mathbf{B}_{ij} \bar{\ell}_{Li} \gamma^\mu P_L \nu_{Lj} + h.c., \end{aligned} \quad (13)$$

where  $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$  and

$$\mathbf{B}_{ij} \equiv \sum_{k=1}^{n_G} \mathbf{U}_{ki}^{\ell_L^*} \mathbf{U}_{kj} \quad (14)$$

is a rectangular  $n_G \times (n_G + n_R)$  matrix.

The main feature distinguishing Dirac and Majorana cases is the existence of *nondiagonal*  $Z\nu_i\nu_j$  vertices (flavour-changing neutral currents), *coupling both left- and right-handed* components of the Majorana neutrinos to the  $Z$  boson,

$$\begin{aligned} -\mathcal{L}_{NC}^Z &= \frac{g}{2c_W} Z_\mu [\bar{\nu}_{Li}^0 \gamma^\mu P_L \nu_{Li}^0 - \bar{\nu}_{Li}^{0c} \gamma^\mu P_R \nu_{Li}^{0c}] \\ &= \frac{g}{2c_W} Z_\mu \bar{\nu}_i [\mathbf{C}_{ij} P_L - \mathbf{C}_{ij}^* P_R] \nu_j \end{aligned} \quad (15)$$

with

$$\mathbf{C}_{ij} \equiv \sum_{k=1}^{n_G} \mathbf{U}_{ki}^* \mathbf{U}_{kj}, \quad (i, j = 1, \dots, n_G + n_R) \quad (16)$$

a quadratic  $(n_G + n_R)^2$  matrix. Such vertices appear in graphs where a  $W$  or a Goldstone boson  $\phi$  is exchanged:

$$\begin{aligned} v_W(i, j) &= -\mathbf{C}_{ij} [\lambda_Q (\mathbb{C}_0 + \mathbb{C}_{11} + \mathbb{C}_{12} + \mathbb{C}_{23}) \\ &\quad - 2\mathbb{C}_{24} + 1] \\ &\quad + \mathbf{C}_{ij}^* \sqrt{\lambda_i \lambda_j} \mathbb{C}_0, \end{aligned} \quad (17)$$

$$\begin{aligned} v_\phi(i, j) &= -\mathbf{C}_{ij} \frac{\lambda_i \lambda_j}{2} \mathbb{C}_0 \\ &\quad + \mathbf{C}_{ij}^* \frac{\sqrt{\lambda_i \lambda_j}}{2} \left[ \lambda_Q \mathbb{C}_{23} - 2\mathbb{C}_{24} + \frac{1}{2} \right]. \end{aligned} \quad (18)$$

The (diagonal) contributions of these graphs for Dirac virtual neutrinos  $[v_W(i) \text{ and } v_\phi(i, j)]$  are obtained by the replacements:

$$\mathbb{C}_{..} \equiv \mathbb{C}_{..}(\lambda_i, \lambda_j) \rightarrow \mathbb{C}_{..}(\lambda_i, \lambda_i), \quad (19)$$

$$\mathbf{C}_{ij} \rightarrow (v_i + a_i) \delta_{ij} = \delta_{ij}, \quad (20)$$

$$\mathbf{C}_{ij}^* \rightarrow -(v_i - a_i) \delta_{ij} = 0. \quad (21)$$

The rest of the contributions are:

$$v_{WW}(i) = 2c_W^2 (2I_3^{iL}) [\lambda_Q (\bar{\mathbb{C}}_{11} + \bar{\mathbb{C}}_{12} + \bar{\mathbb{C}}_{23}) - 6 \bar{\mathbb{C}}_{24} + 1], \quad (22)$$

$$v_{\phi\phi}(i) = -(1 - 2s_W^2) (2I_3^{iL}) \lambda_i \bar{\mathbb{C}}_{24}, \quad (23)$$

$$v_{W\phi}(i) = -2s_W^2 (2I_3^{iL}) \lambda_i \bar{\mathbb{C}}_0, \quad (24)$$

$$v_\Sigma(i) = \frac{1}{2}(v_i + a_i - 4c_W^2 a_i) \times [(2 + \lambda_i)\mathbb{B}_1 + 1]. \quad (25)$$

We have introduced above dimensionless two- and three-point one-loop functions:

$$\mathbb{B}_1 \equiv \mathbb{B}_1(\lambda_i) = B_1(0; m_i^2, M_W^2), \quad (26)$$

$$\bar{\mathbb{C}}_{..} \equiv \bar{\mathbb{C}}_{..}(\lambda_i) = M_W^2 C_{..}(0, Q^2, 0; m_i^2, M_W^2, M_W^2), \quad (27)$$

$$\mathbb{C}_{..} \equiv \mathbb{C}_{..}(\lambda_i, \lambda_j) = M_W^2 C_{..}(0, Q^2, 0; M_W^2, m_i^2, m_j^2), \quad (28)$$

from the usual tensor integrals [15]

$$B^\mu(p^2; m_0^2, m_1^2) = p^\mu B_1, \quad (29)$$

$$C^\mu(p_1^2, Q^2, p_2^2; m_0^2, m_1^2, m_2^2) = p_1^\mu C_{11} + p_2^\mu C_{12}, \quad (30)$$

$$C^{\mu\nu}(p_1^2, Q^2, p_2^2; m_0^2, m_1^2, m_2^2) = p_1^\mu p_1^\nu C_{21} + p_2^\mu p_2^\nu C_{22} + (p_1^\mu p_2^\nu + p_2^\mu p_1^\nu) C_{23} + g^{\mu\nu} M_W^2 C_{24}. \quad (31)$$

The tensor integrals are numerically evaluated with the computer program **LoopTools** [16].

The expressions above, in the Dirac case, are also valid for quark flavour-changing  $Z$  decays involving virtual quarks  $i$ , just by substituting their corresponding weak isospin  $I_3^{iL}$ , electric charge  $e_i$ , and vector  $v_i$  and axial-vector  $a_i$  couplings,

$$v_i = I_3^{iL} - 2e_i s_W^2, \quad (32)$$

$$a_i = I_3^{iL}. \quad (33)$$

It turns out convenient to cast (11) as

$$V_M(i, j) = \delta_{ij} F(\lambda_i) + \mathbf{C}_{ij} G(\lambda_i, \lambda_j) + \mathbf{C}_{ij}^* \sqrt{\lambda_i \lambda_j} H(\lambda_i, \lambda_j). \quad (34)$$

The Dirac form factor (9) is then

$$V(i) = F(\lambda_i) + G(\lambda_i, \lambda_i). \quad (35)$$

The amplitude  $\mathcal{M}$  is finite without renormalization, but the form factors  $V$  and  $V_M$  are not. The divergencies are such that they exactly cancel due to unitarity relations among the mixing matrix elements  $\mathbf{V}$ ,  $\mathbf{B}$  and  $\mathbf{C}$  [9,14,11].

The branching ratio  $Z \rightarrow \ell_1^\mp \ell_2^\pm$  reads

$$\text{BR}(Z \rightarrow \ell_1^\mp \ell_2^\pm) = \frac{\alpha_W^3 M_Z}{192 \pi^2 c_W^2 \Gamma_Z} |\mathcal{V}(M_Z^2)|^2. \quad (36)$$

The constant in front of (36) is of  $\mathcal{O}(10^{-6})$ .

### 3. Contribution from light neutrinos: $\nu\text{SM}$

Ignoring the not yet confirmed results of the LSND accelerator experiment, all neutrino experiments are compatible with the oscillation between two of a total of three neutrino species. The signals in atmospheric experiments [17] are at the 90% c.l. compatible with  $\nu_\mu - \nu_\tau$  oscillations,

$$\Delta m_{\text{atm}}^2 = \Delta m_{23}^2 \simeq (2 \div 8) \times 10^{-3} \text{eV}^2, \quad (37)$$

$$\sin^2 2\theta_{\text{atm}} = \sin^2 2\theta_{23} \simeq 0.82 \div 1.0. \quad (38)$$

Solar experiments [18] indicate  $\nu_e - \nu_\mu$  mixing,

$$\Delta m_\odot^2 = \Delta m_{12}^2 \simeq 10^{-10} \div 10^{-5} \text{eV}^2, \quad (39)$$

$$\sin^2 2\theta_\odot = \sin^2 2\theta_{12} = \text{free} \quad (40)$$

(there are solutions for vacuum and matter oscillations compatible with a wide range of mixing angles). From reactor searches, there are no hints of  $\nu_e - \nu_\tau$  oscillations [19], which implies

$$\sin^2 2\theta_{13} = 0. \quad (41)$$

Taking this information into the standard parametrization for the mixing matrix [2] (oscillation experiments are insensitive to Majorana CP-phases) and putting the Dirac CP-phase  $\delta = 0$ , since no information on it is yet available, one has

$$\mathbf{V} \simeq \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{1}{\sqrt{2}}s_{12} & \frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}s_{12} & -\frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (42)$$

Using the unitarity of  $\mathbf{V}$ , with  $\ell_1 \neq \ell_2$ ,

$$\text{BR}(Z \rightarrow \ell_1^\mp \ell_2^\pm) = \frac{\alpha_W^3 M_Z}{192\pi^2 c_W^2 \Gamma_Z} \times \left| \sum_{i=1}^3 \mathbf{V}_{\ell_1 i} \mathbf{V}_{\ell_2 i}^* [V(\lambda_i) - V(0)] \right|^2. \quad (43)$$

Performing a low neutrino mass expansion of the tensor integrals ( $\lambda_i \ll 1$ ) one finds [20,21]

$$V(\lambda_i) - V(0) = a_1 \lambda_i + \mathcal{O}(\lambda_i^2), \quad (44)$$

$$a_1 = 2.5623 - 2.950 i. \quad (45)$$

Therefore  $\text{BR}(Z \rightarrow \ell_1^\mp \ell_2^\pm)$  goes as  $m_i^4$  for low neutrino masses. This behaviour is still a good approximation not far below the  $Z$  mass (see Fig. 1).

Substituting the phenomenological squared mass differences  $\lambda_{ij} \equiv \Delta m_{ij}^2 / M_W^2$  and the mixing angles, one has [11]

$$\begin{aligned} \text{BR}(Z \rightarrow e^\mp \mu^\pm) &\simeq \text{BR}(Z \rightarrow e^\mp \tau^\pm) \\ &\approx 6 \times 10^{-6} \times c_{12}^2 s_{12}^2 \lambda_{12}^2 \\ &\lesssim 4 \times 10^{-60}, \end{aligned} \quad (46)$$

$$\begin{aligned} \text{BR}(Z \rightarrow \mu^\mp \tau^\pm) &\simeq 3 \times 10^{-6} \times |s_{12}^2 \lambda_{12} - \lambda_{23}|^2 \\ &\approx (3 \div 30) \times 10^{-55}. \end{aligned} \quad (47)$$

These rates are extremely small. In fact, the contributions from the observed light neutrinos will be neglected in the next section, where we extend the  $\nu\text{SM}$  to accomodate heavy neutrinos, taking massless the light ones.

#### 4. Contribution from heavy neutrinos

Assume that there is a sector of heavy neutrinos  $N_i$  mixing with the light ones. Using a general formalism developed in [22] one can exploit measurements of flavour diagonal processes (checks of lepton universality and CKM unitarity,  $Z$  boson invisible width, ...) [23] to obtain *indirect* experimental bounds on light-heavy mixings [14]:

$$s_{\nu_\ell}^2 \equiv \left| \sum_i \mathbf{B}_{\ell N_i} \right|^2 \quad (48)$$

(replacing  $\mathbf{B}$  by  $\mathbf{V}$  for Dirac neutrinos). The most recent indirect bounds [24]:

$$s_{\nu_e}^2 < 0.012, \quad s_{\nu_\mu}^2 < 0.0096, \quad s_{\nu_\tau}^2 < 0.016, \quad (49)$$

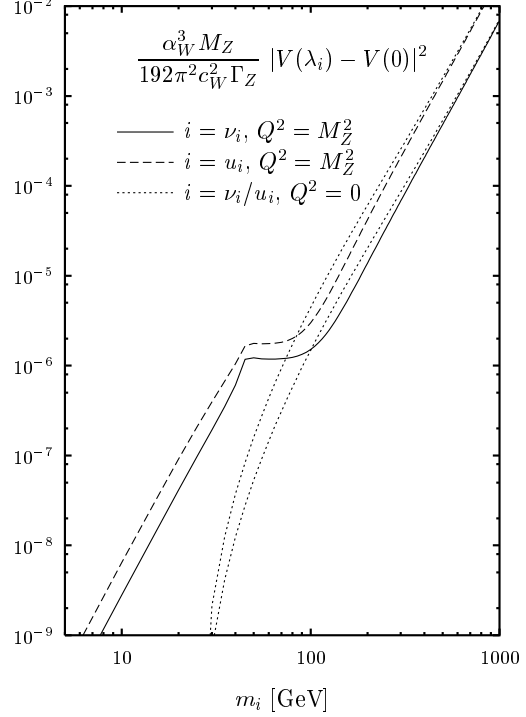


Figure 1. Contribution of one massive Dirac neutrino (solid) or up-quark (dashed) to BR normalized to unit mixing. Dotted lines using  $Q^2 = 0$ .

are only improved by *direct* searches for flavour nondiagonal processes involving the first two lepton generations. In fact, from  $\text{BR}(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}$  [25] one may infer [26]:

$$s_{\nu_e} s_{\nu_\mu} < 1.4 \times 10^{-8}. \quad (50)$$

##### 4.1. $\nu\text{SM}$ + one heavy ordinary Dirac

From unitarity of  $\mathbf{V}$  for  $n_G + 1$  generations,

$$\begin{aligned} \text{BR}(Z \rightarrow \ell_1^\mp \ell_2^\pm) &= \frac{\alpha_W^3 M_Z}{192\pi^2 c_W^2 \Gamma_Z} \left| \mathbf{V}_{\ell_1 N} \mathbf{V}_{\ell_2 N}^* \right|^2 \\ &\times |V(\lambda_N) - V(0)|^2. \end{aligned} \quad (51)$$

The form factor  $V$ , subtracted, squared and normalized is depicted in Fig. 1. The results agree with earlier calculations [9], also for the quark flavour-changing  $Z$  decays [10]. As expected, the

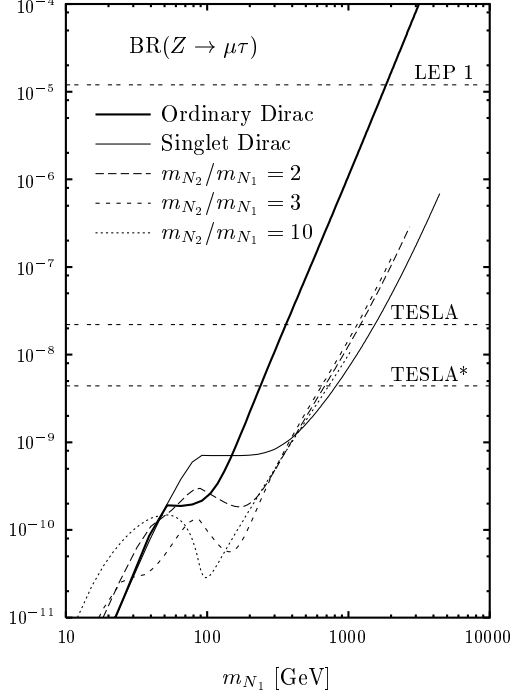


Figure 2. Maximal value of the branching ratio of  $Z \rightarrow \mu^\mp \tau^\pm$  in the  $\nu$ SM extended with: (i) one heavy ordinary (thick solid) or singlet (thin solid) Dirac neutrino of mass  $m_{N_1}$ ; (ii) two heavy right-handed singlet Majorana neutrinos (dashed lines) with masses  $m_{N_1}$  and  $m_{N_2}$ . The upper limits of (49) are taken as light-heavy mixings.

approximation  $Q^2 = 0$  is very bad for  $m_N \lesssim M_Z$  but it makes sense for  $m_N \gg M_Z$  (Fig. 1).

Taking the present upper bounds of the mixing matrix elements from (48,49), Fig. 2 shows the maximal  $\text{BR}(Z \rightarrow \mu^\mp \tau^\pm)$ , for illustration.

It is worth noticing that the expansion of tensor integrals in the large mass limit [9,11] at the  $Z$  peak yields

$$V(\lambda_N) - V(0) = \frac{1}{2}[\lambda_N + 2.88 \ln \lambda_N - (6.99 + 2.11 i)] + \mathcal{O}(\ln \lambda_N / \lambda_N), \quad (52)$$

leading again to an  $m_N^4$  growth of the branching ratios for large neutrino masses.

#### 4.2. $\nu$ SM + ( $n_R = 2$ ) Majorana neutrinos

Unitarity constraints on  $\mathbf{B}$  and  $\mathbf{C}$  [14] allow to write  $\mathcal{V}_M$  in terms of the heavy sector only:

$$\begin{aligned} \mathcal{V}_M = & \sum_{i,j=1}^{n_R} \mathbf{B}_{\ell_1 N_i} \mathbf{B}_{\ell_2 N_j}^* \\ & \times \left\{ \delta_{N_i N_j} [F(\lambda_{N_i}) - F(0) + G(\lambda_{N_i}, 0) \right. \\ & \quad \left. + G(0, \lambda_{N_i}) - 2G(0, 0)] \right. \\ & + \mathbf{C}_{N_i N_j} [G(\lambda_{N_i}, \lambda_{N_j}) - G(\lambda_{N_i}, 0) \\ & \quad \left. - G(0, \lambda_{N_j}) + G(0, 0)] \right. \\ & \left. + \mathbf{C}_{N_i N_j}^* \sqrt{\lambda_{N_i} \lambda_{N_j}} H(\lambda_{N_i}, \lambda_{N_j}) \right\}^2. \quad (53) \end{aligned}$$

For  $n_R = 2$  the mixing matrices are exactly calculable in terms of  $s_{\nu_\ell}^2$  and  $r \equiv m_{N_2}^2/m_{N_1}^2$  [14]. The upper values for the branching ratios can be then straightforwardly obtained from the bounds (49), given the heavy masses  $m_{N_1}$ ,  $m_{N_2}$  (Fig. 2). The case  $m_{N_1} = m_{N_2}$  is equivalent to *one heavy singlet Dirac* neutrino (in fact, two equal mass Majorana neutrinos with opposite CP parities form a Dirac neutrino).

In the large neutrino mass limit ( $\lambda_{N_1} \gg 1$ ) one obtains [21]

$$\begin{aligned} \mathcal{V}_M(\lambda_{N_1}, r; s_{\nu_\ell}) = & s_{\nu_{\ell_1}} s_{\nu_{\ell_2}} \\ & \times \left\{ \frac{\sum_\ell s_{\nu_\ell}^2}{(1 + r^{\frac{1}{2}})^2} \left( \frac{3}{2}r + \frac{r^2 + r - 4r^{\frac{3}{2}}}{4(1 - r)} \ln r \right) \lambda_{N_1} \right. \\ & \left. + \frac{1}{2} \left( 3 + \frac{1 - 2c_W^2}{6} \lambda_Q \right) \ln \lambda_{N_1} \right\} + \mathcal{O}(1). \quad (54) \end{aligned}$$

The constant in front of the  $\ln \lambda$  term is identical to the one in the Dirac case (52). In the approximation  $\lambda_Q \equiv Q^2/M_W^2 = 0$ , the expression (54) is in agreement with [14], but we take the actual value  $\lambda_Q = M_Z^2/M_W^2 = 1/c_W^2 \approx 1.286$  in the whole calculation.

Finally, notice that large neutrino masses are restricted by the perturbative unitarity condition on the decay width of heavy neutrinos [27],

$$\Gamma_{N_i} \simeq (2 \times) \frac{\alpha_W}{8M_W^2} m_{N_i}^3 \sum_\ell |\mathbf{B}_{\ell N_i}|^2 \leq \frac{1}{2} m_{N_i} \quad (55)$$

for Dirac (Majorana) neutrinos. In this way, the unacceptable large-mass behaviour of the amplitudes ( $\propto m_N^2$ ) is actually cured when a sensible

light-heavy mixing (at most  $\propto m_N^{-2}$ ) is taken into account [28]. The restrictions (55) lead to the end-points in the curves of Fig. 2.

## 5. Summary and conclusions

The perturbative unitarity limit on the decay width of heavy neutrinos effectively prevents their nondecoupling, thus ensuring smaller light-heavy mixings for increasing heavy masses. Given the present *indirect* upper bounds on light-heavy mixings of  $\mathcal{O}(10^{-2})$ , this already sets indirect upper limits on LFV  $Z$  decays of about  $\text{BR} \lesssim 10^{-6}$  with Majorana neutrinos and above LEP 1 reach for Dirac neutrinos.

We have presented the full one-loop expectations for the *direct* lepton flavour changing process  $Z \rightarrow \ell_1 \ell_2$  with virtual Dirac or Majorana neutrinos. We conclude that: (i) the contributions from the observed light neutrino sector are far from experimental verification ( $\text{BR} \lesssim 10^{-54}$ ); (ii) the Giga- $Z$  mode of the future Tesla linear collider, sensitive down to about  $\text{BR} \lesssim 10^{-8}$ , might have a chance to produce such processes.

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